A MultiGaussian Approach to Assess Block Grade Uncertainty

Julián M. Ortiz[1], Oy Leuangthong[2], and Clayton V. Deutsch[2]

[1]Department of Mining Engineering,
University of Chile
[2]Department of Civil & Environmental Engineering,
University of Alberta

Abstract

Uncertainty quantification of a spatially distributed variable at any scale can be handled through geostatistical simulation. Large computation time, storage, and post-processing of the realizations are required to obtain a final assessment of block uncertainty.

Multi-Gaussian kriging is a flexible alternative to simulation. The idea is to compute the conditional distribution of uncertainty after normal score transformation of the original samples. Under the multi-Gaussian assumption, all marginal and conditional distributions are Gaussian, hence fully defined by their mean and variance. The parameters of the conditional distributions are obtained by simple kriging and can be back-transformed to the original units of the variable of interest. An estimate and any summary of uncertainty can be easily retrieved.

The main disadvantage of performing multi-Gaussian kriging is that change of support is not straightforward, that is, calculating uncertainty of block grades. We propose a methodology to overcome this limitation by considering a matrix simulation to generate multiple probability fields. Each probability field is used to draw spatially correlated point values from the point-support conditional distributions, and multiple realizations of the average can be obtained. This permits the calculation of the average over the block and its uncertainty. These blocks may correspond to selective mining units or to volumes from longer production periods relevant for engineering decisions. They can even be disjoint blocks, such as when several faces are mined at the same time. POSTMG, a Fortran program to perform these calculations, is described and a case study is provided.

Introduction

Managing risk in mining projects requires assessing the uncertainty associated with production estimates. These estimates and their uncertainty can be computed through geostatistical simulation by creating multiple numerical models of the distribution of block grades that honour the local data and reproduce their spatial variability, and then assessing the variability in the average grade for the production period. The uncertainty in grade for monthly, quarterly, and yearly production helps in mine plan design to ensure a given metal production is quantified. When the models are large, computing time and storage may prohibit the use of simulation. This paper proposes an alternative approach to assess grade uncertainty over block volumes.

Estimation methods such as indicator kriging and multi-Gaussian kriging allow assessment of uncertainty of the point grades, that is, we can define the probability of a point to have a grade large than a cutoff. However, determining the block grade uncertainty from the point grade uncertainty is not straightforward. We propose the local use of probability field simulation to account for the correlation existing between the points in the production volume of interest. The simulation works by generating multiple outcomes of the average grade over the volume and building its distribution of uncertainty. Confidence intervals can be computed and these can be used to define levels of certainty about the planned mineral production.

The paper is organized as follows: we briefly review multi-Gaussian kriging as a means to calculate point grade uncertainty; then, we discuss the approaches to evaluate the block grade uncertainty; some methodological and implementation details are discussed; and a case study is presented to illustrate the methodology.

MultiGaussian Kriging

Kriging is a well known technique to generate estimates of point and block grade values. However, kriging does not provide a measure of the uncertainty about the estimate. The kriging variance could be seen as a measure of the reliability of the estimate, however, it is independent of the data values and, in reality, the grades often show a *proportional effect*, that is, a relationship between the local grade and its variability (Isaaks and Srivastava, 1989; Journel and Huijbregts, 1978). Furthermore, one would need to assume the shape of the distribution of uncertainty to allow calculation of confidence intervals.

MultiGaussian kriging amounts to simple kriging applied to a Gaussian or normal transformation of the original sample data. The original distribution of sample grades is converted into a standard normal (or Gaussian) distribution. This is the normal score transformation shown in Figure 1 (Verly, 1983; Verly, 1984). This transformation is made because, under the assumption of multivariate Gaussianity, the shape of the univariate distribution of grades must be Gaussian. All conditional distributions are Gaussian with mean and variance calculated through simple kriging.

The transformation requires a representative distribution of the original sample grades, hence declustering is performed. Declustering corrects for the bias on the statistics of the distribution originated from sampling preferentially the high grade locations. Calculation and modelling of the normal scores variogram is then required (Deutsch and Journel, 1997).

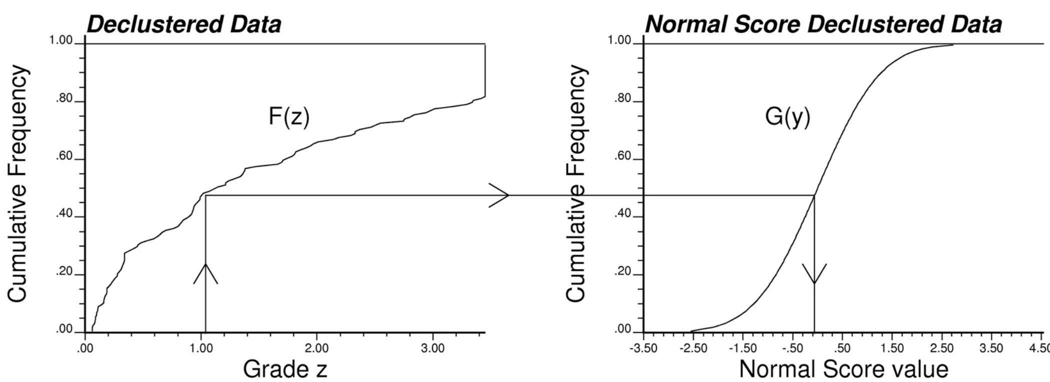

Figure 1: The normal score transformation is illustrated for a data z_i. The cumulative frequency is read in the original distribution and the value y_i of a standard normal distribution, that is, a Gaussian distribution with mean 0 and variance 1, corresponding to that cumulative frequency is assigned to the data location.

MultiGaussian kriging can then be performed to obtain the mean and variance of the transformed variable, which is sufficient to define the full conditional distribution in Gaussian units. The full conditional distribution in the original units of the variable can then be retrieved by back-calculating the z values for given percentiles. The mean can be back-calculated by numerical integration, as illustrated in Figure 2. In theory, the available information allows us to calculate the estimate with minimum estimation variance by simple kriging.

One of the advantages of transforming the variable to a Gaussian distribution is that it "filters" the proportional effect. The relationship between the local mean and local standard deviation shown by the original grades vanishes once the variable is transformed into a standard normal distribution. Figure 3 illustrates the relationship between local means and standard deviations of the original grades and the normal scores of the grades, calculated with a moving window of 100 x 100 x 100 m^3. The linear relationship that exists in the original variable disappears when the transformed variable is considered. Once the local point grade distributions are calculated, these can be back-transformed to original units, thereby re-injecting the existing proportional effect. Multi-Gaussian kriging provides an easy approach to calculate the conditional distributions, with the same requirements, in terms of inference, as Gaussian simulation: a representative histogram is transformed and the variogram of the normal scores is calculated and modelled.

Simulation for Block Uncertainty

Assessing block uncertainty can be done using geostatistical simulation methods. Sequential methods work by randomly visiting all the uninformed nodes in the simulation grid. The point distribution of uncertainty is computed at each location given the sample data and previously simulated nodes, and a value is drawn from this distribution using Monte-Carlo simulation. This value is used for all subsequent nodes as conditioning data; this ensures reproduction of the spatial correlation (Journel, 1993). Due to the requirement to include previously simulated points to condition subsequent nodes in the random path, storage may become an issue if very large grids are considered (over ten million nodes). Furthermore, the time required to compute multiple realizations may also be a problem. If N nodes are being simulated for L realizations, then NL kriging systems must be solved. Since the simulated points are also used to condition the estimation in subsequent nodes, the kriging matrices tend to be large in size, increasing the computer time required to invert them. In estimation, only N kriging systems must be solved and these are in general not as large as the ones considered in the case of simulation, and storage is required only for the sample data, which are the only data used for conditioning.

The typical procedure to assess uncertainty over block values using simulation is:

- L dense grid simulated realizations are generated.
- Block values are calculated for each one of the L realizations.
- The L block values are used to construct the distribution of uncertainty (histogram) for the block variable at a specific location.
- Mean and any quantile of the block distribution can be retrieved from the set of L block values.

Simulation may not be practical if a very large block model needs to be built. The storage required to keep the realizations and time necessary to build and post-process them be prohibitive, hence an alternative approach may be considered.

Proposed Methodology

An alternative to simulation is to use the local distributions of uncertainty to infer the block support uncertainty. In order to combine the point support uncertainty distributions into a single block distribution, the spatial correlation must be taken into account. Consider for example, the case where perfect correlation exists (Figure 4, left). In this case, the point grade distribution is constant for all locations. The uncertainty on the average over several locations is then equal to the point grade uncertainty. A second extreme case corresponds to a pure nugget effect (Figure 4, right). Assuming all points have the same distribution of uncertainty, the average over several points will have a smaller uncertainty than in the case of perfect correlation.

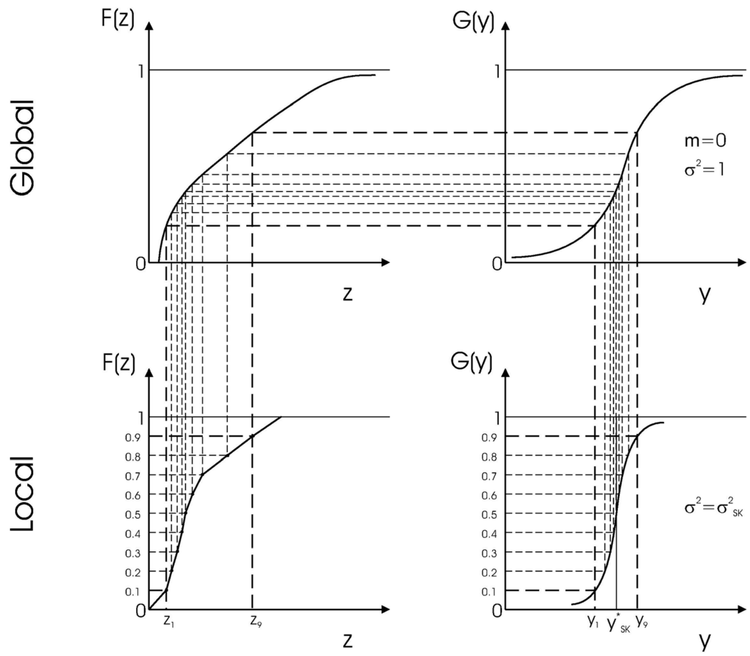

Figure 2: Calculation of the mean by numerical integration. The local uncertainty distribution is given by the kriging estimate and variance and the assumption that the shape is normal (bottom right). Several quantiles are calculated in the illustration. The nine deciles of the distribution, $y_1,...,y_9$, are back-transformed (top) and the corresponding values, $z_1,...,z_9$, are averaged to calculate the mean (bottom left).

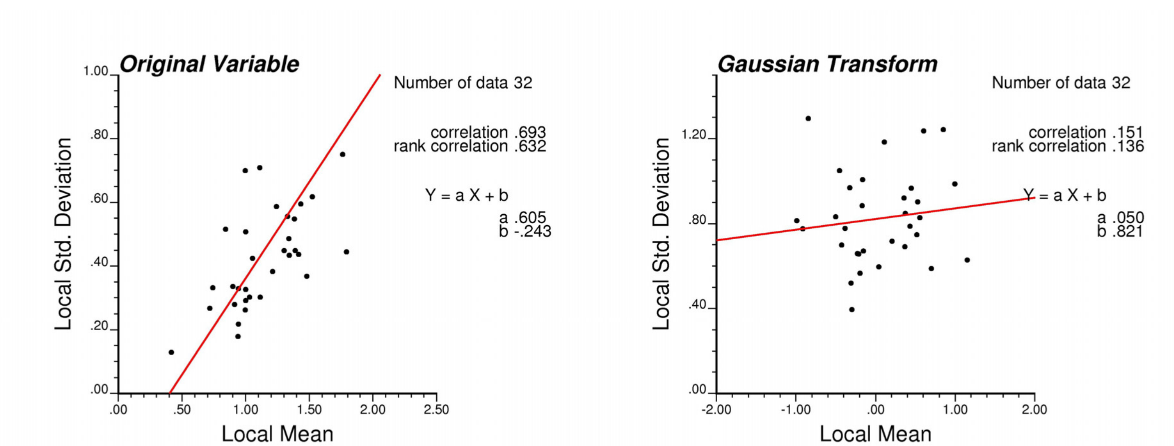

Figure 3: Proportional effect before and after transforming the grade values to a standard normal distribution. A local window of 100 x 100 x 100 m^3 was considered to calculate the local statistics.

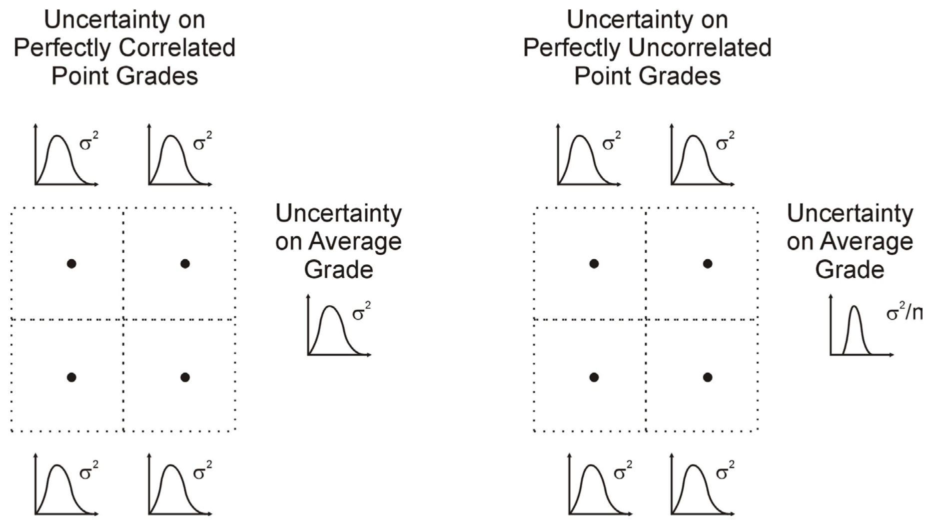

Figure 4: Two extreme cases for the uncertainty on the average grade. On the left, the case of perfect correlation between the point grades generates a distribution of uncertainty for the average grade with a variance as in the point distributions. On the right, the points are uncorrelated, generating a smaller variance for the average.

Probability field (p-field) simulation provides a fast method to combine these point conditional distributions into a block distribution by generating spatially correlated values from the point distributions (Froidevaux, 1993; Srivastava, 1992). The entire procedure is illustrated in Figure 5. The point scale uncertainty is determined by multiGaussian kriging; several probability fields are generated such that the probability value at one location is correlated with the probability value at a nearby location; each probability field is used to draw from the conditional distribution a simulated value such that a proportion p of the values lays below it; the average simulated grade for each block can be determined by averaging the simulated point grades, from the set of realizations, a histogram of simulated block grades can be built from which any summary statistic of its uncertainty can be retrieved.

Furthermore, since the p-field is used to draw from the conditional distributions, it can be generated unconditionally, hence a fast algorithm such as spectral methods or matrix decomposition can be used.

The following methodology is proposed to assess block uncertainty:

1. Calculate the multi-Gaussian kriging estimate and variance at point support by kriging the normal score data using the corresponding variogram of normal scores.
2. Define the blocks of interest for uncertainty calculation, based on the objective of the study:
    a. Regular blocks can be defined for the calculation of recoverable reserves or for resource or reserve classification.
    b. A production volume (irregular shape and probably disconnected) can be considered for assessing uncertainty in average grade for a planned production period.
3. For every block of interest, retrieve the points located within the block and their locations.
4. Generate multiple unconditional probability fields of the points located inside each block, one block at a time, using a matrix method, with the normal score variogram.
5. For every probability field:

   a. Generate the simulated values by drawing from the conditional distributions using the probability field.
   b. Back-transform every simulated value to the original units and average the values in original units to obtain a single realization of the average.
6. Pool together the average values for a particular block, and retrieve its variance or any probability interval as a measure of uncertainty.

This methodology has been implemented and applied to a case study presented in the next section.

Case Study: Porphyry Copper Deposit

The objective of this study is to show the implementation of the method proposed for calculating the uncertainty at block support from point grade uncertainties. A drillhole database with copper grades in 12m composites from a porphyry copper mine is available for this study. The data base contains East, North and elevation coordinates, and the grade in percent by weight.

Figure 6 shows the declustered histogram of composites. 1281 samples are available. The data range from 0 to approximately 7 % Cu and the distribution is positively skewed. The coefficient of variation is approximately 0.5, which can be considered relatively low. It is a typical value for deposits of this type. The median is very close to the mean value.

A plan view of the drillholes at a particular bench is shown in Figure 7. The copper grades are shown at the bench level with a tolerance of 12 m. The average spacing between drillholes is around 50 m. In many zones drillholes are spaced even closer.

Variogram of Normal Scores

The variogram of the normal scores is calculated and modelled. The final model consists of a 20% nugget effect, a spherical structure that contributes 15% to the total sill and an exponential structure that contributes with the remaining 65% of the sill. The ranges are shown in Table 1 and the experimental and fitted variogram in the three main directions of anisotropy are shown in Figure 8.

Multi-Gaussian Kriging

Multi-Gaussian kriging is performed, that is, the normal scores of the data are used as conditioning information to obtain kriging estimates and variances at point support within the domain of interest. The grid simulated is defined in Table 2.

The search strategy was defined considering 4 and 16 samples as minimum and optimum numbers for estimating a location. The search ellipsoid was defined with its main axis rotated to N30ºE. The maximum, minimum and vertical radii that define the ellipsoid were 160, 110, and 220 m.

Figure 9 shows the estimates and variances in transformed units.

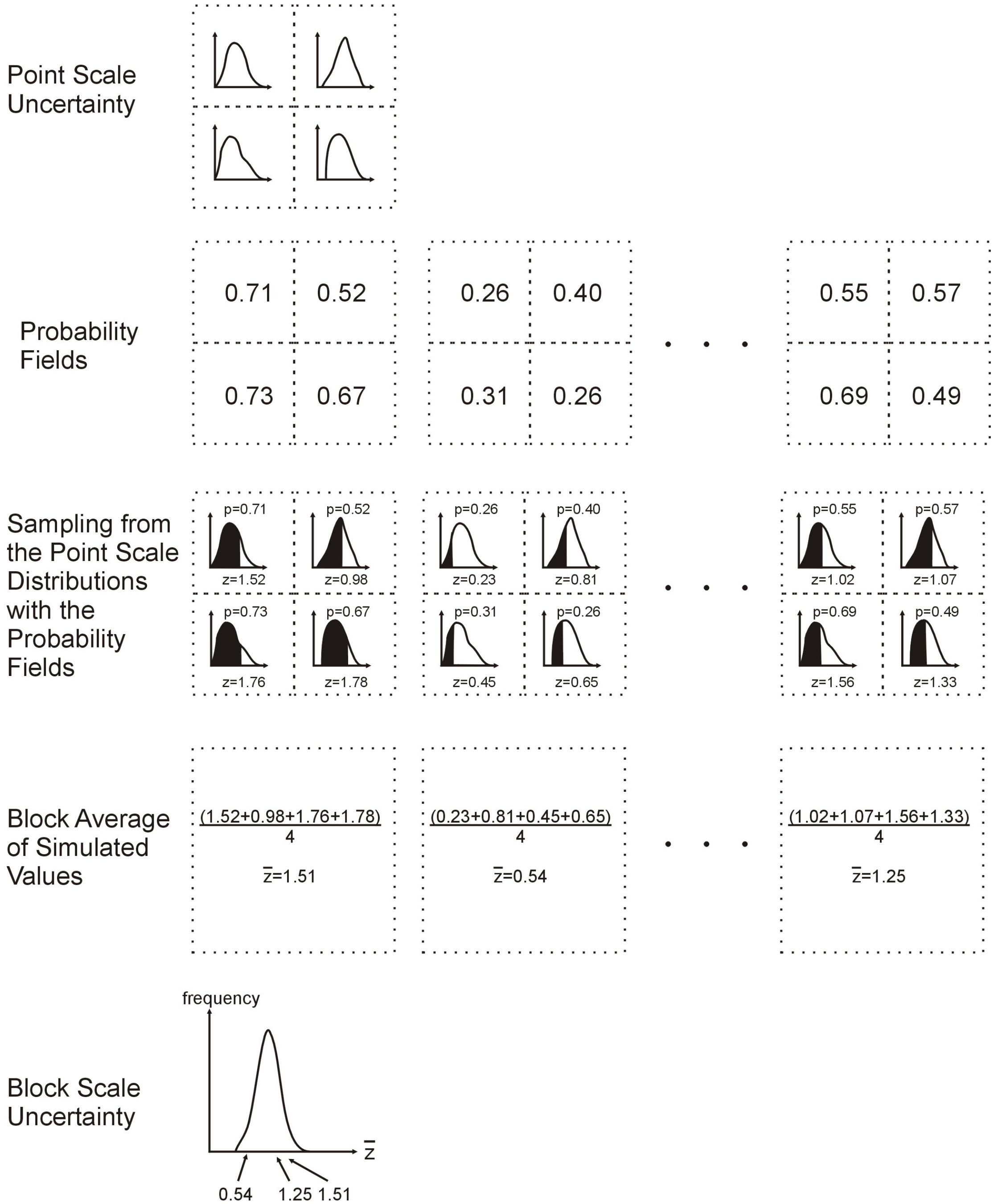

Figure 5: P-field simulation: starting from the point scale uncertainty distributions of the points within the block, multiple simulated grade values at point scale are generated, by drawing using the probabilities of a set of p-fields. These probability fields are generated independently as unconditional realizations of a multiGaussian correlated random variable. They inject the correlation by drawing from the conditional distributions with correlated p-values, rather than randomly, as it is done in Monte-Carlo simulation. The simulated grades are then averaged within each realization to obtain a simulated block grade. The set of block grades generated from the set of realizations is used to build a histogram that represents the block grade uncertainty.

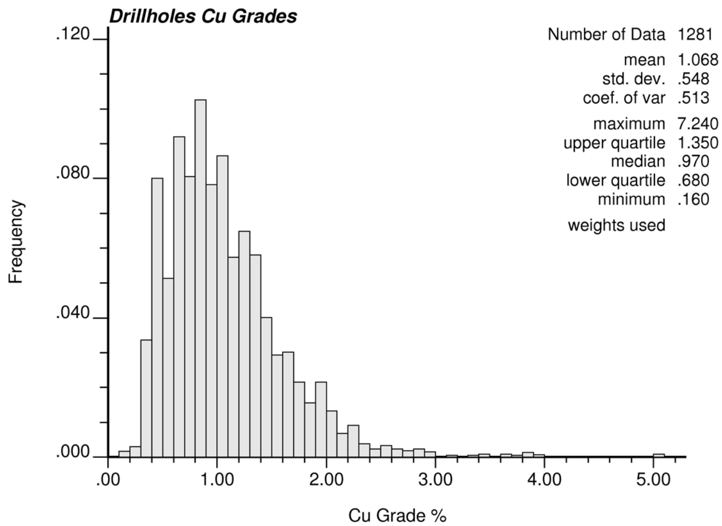

Figure 6: Declustered histogram of copper grades**.**

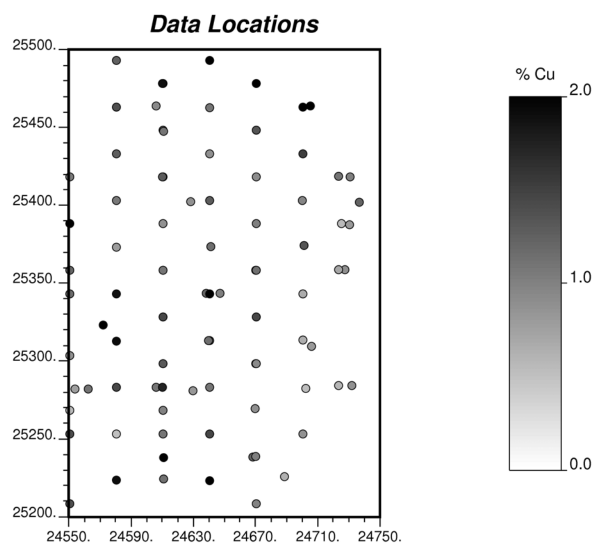

Figure 7: Plan view showing the drillhole information for bench 3922.

	Ranges (m)	
Directio n	Spherical Structure	Exponential Structure
N 30ºW	20.0	160.0
N 60ºE	60.0	105.0
Vertical	45.0	220.0

Table 1: Ranges of modelled variogram structures.

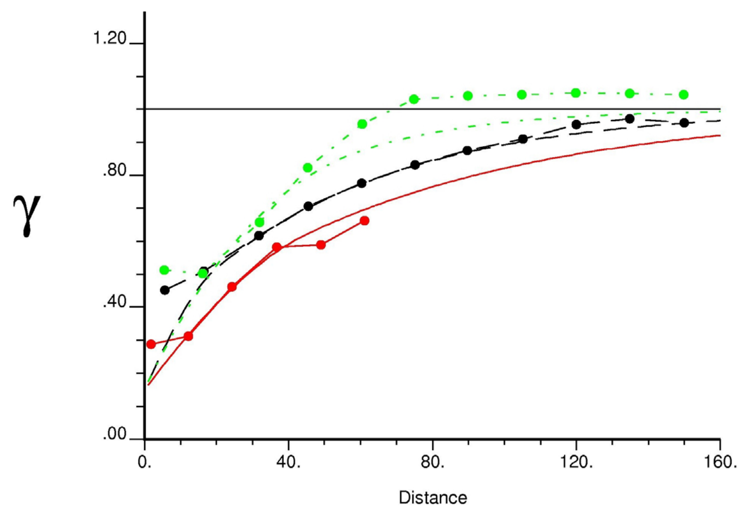

Figure 8: Normal scores variogram model. The solid line corresponds to the vertical direction, the dashed line is in the N30ºW direction, and the dotted line corresponds to the N60ºE direction.

Direction	Number of Nodes	Coordinate Centre of First Block (m)	Size (m)
Easting	100	24551.0	2.0
Northing	150	25201.0	2.0
Elevation	12	3881.0	2.0

Table 2: Grid definition.

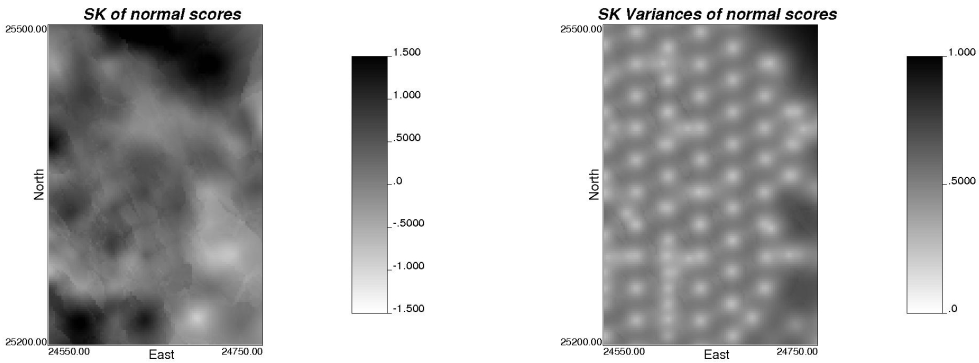

Figure 9: Maps of estimates and variances of normal scores.

Block Distributions

Multi-Gaussian kriging provides an estimate and kriging variance of the normal score value at unsampled locations (Figure 9), which together with the assumption that conditional distributions are Gaussian in shape can be used to obtain an estimate and any confidence interval in original units. Averaging of the point distribution was done for blocks of 10 by 10 by 12 m, using 200 quantiles to perform the numerical integration required. Finally, 100 realizations of the probability fields were considered, from which the simulated point values were drawn, back-transformed and averaged to obtain the distribution of block grades. Figure 10 shows the estimates and variances in original units at point support (top) and at block support (bottom). Notice that the variance does not depend exclusively on the spatial configuration of the data, but also on its local mean. The proportional effect of the original data (see Figure 3) is preserved in the final results due to the Gaussian transformation.

Possible Applications

These block grade distributions of uncertainty could be used to determine recoverable reserves. The proportion of each block with a grade higher than a given cutoff can easily be retrieved. Adding these proportions and calculating the grade of the material above the cutoff grade provides an estimate of the recoverable reserves (without accounting for mining dilution).

A second possible application is the use of the methodology for classification. The program could be modified to calculate locally an estimated distribution of uncertainty for blocks of different sizes. These could represent production for different periods. Blocks could be classified based on their spread around the mean, at a given confidence level.

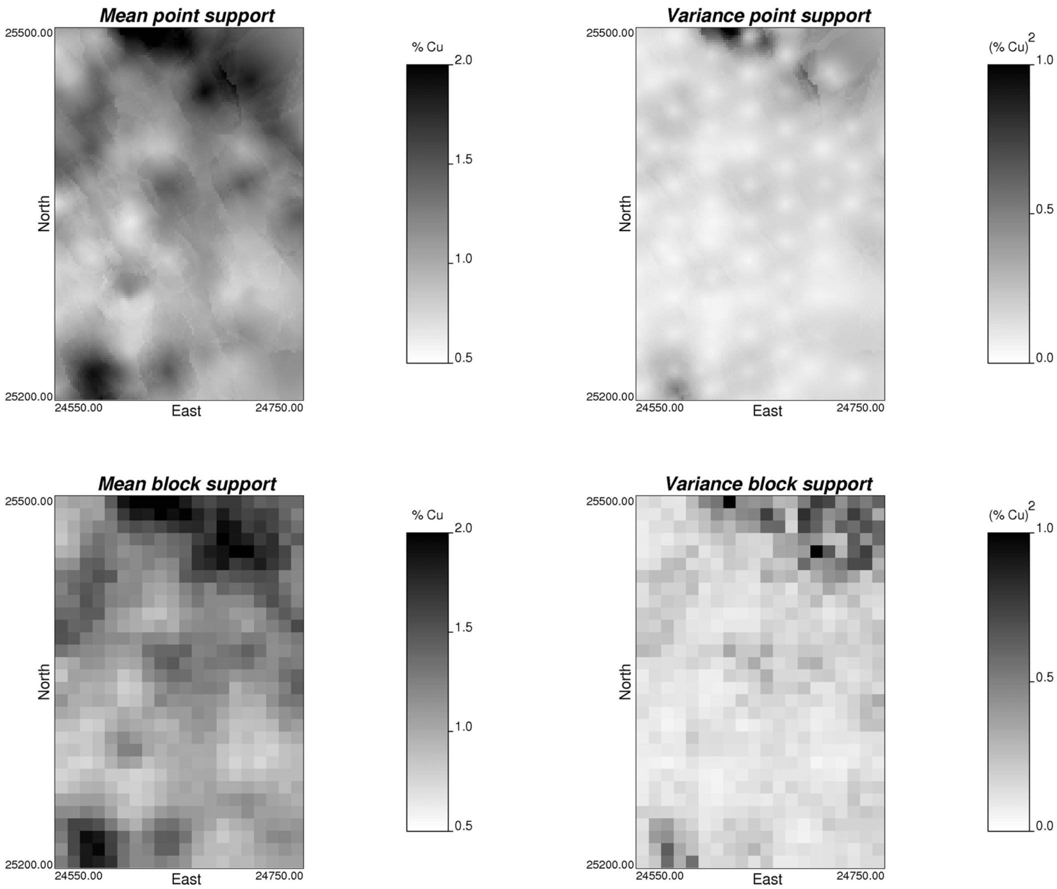

Figure 10: Maps of estimates and variance in original units at point support (top) and at block support (bottom).

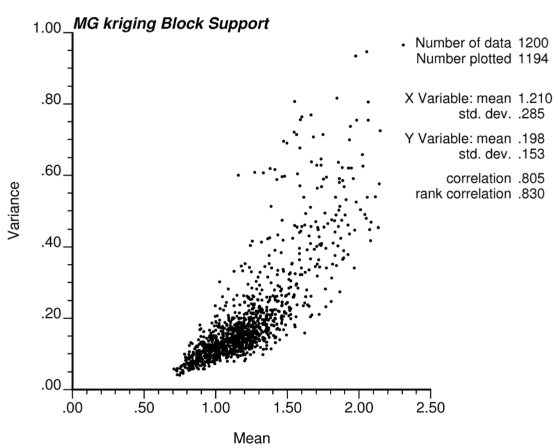

Figure 11: Proportional effect of estimated block grades.

Conclusions

A methodology to post-process the point distributions of uncertainty obtained with multi-Gaussian kriging, to obtain block grade estimates with an associated measure of uncertainty is illustrated. The methodology was implemented for an application to a copper deposit.

The block grade distributions of uncertainty account for the spatial correlation between the points inside the block, via a p-field simulation of the point grades. The correlated probabilities contained in every realization of the probability field are used to draw from the conditional distributions from multi-Gaussian kriging. The simulated point values are back-transformed with the corresponding global transformation table and averages in original units are calculated. These averages can be pooled together into a histogram that represents the block grade uncertainty. Any summary measure of uncertainty can be retrieved from these histograms, and they can be used for classification, production uncertainty assessment and calculation of recoverable reserves.

The proposed methodology represents an alternative to the computationally intensive approach of simulating the entire field repeatedly to retrieve the average grades of blocks within the deposit.

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